DETECTING AND MODELING CHANGING VOLATILITY IN THE COPPER FUTURES MARKET

KEVIN BRACKER
KENNETH L. SMITH*

Copper futures returns are characterized by negative skewness and excess kurtosis. Research has not yet examined this nonnormality, which contributes to their volatility. To date little attention has been paid to the modeling of these series. Therefore, the purpose of this paper is to (i) detect alternating subperiods of volatility by using a method that uses an iterated cumulative sum of squares (ICSS) algorithm to identify breakpoints in the series; and (ii) compare the ability of five models (the random walk, GARCH, EGARCH, AGARCH, and the GJR model) to capture the volatility within each ICSS identified subperiod. These tests were applied to two copper futures series (open to close and close to close prices). Results indicate that the ranking (in terms of the root mean square error) is similar for both series. That is, the GARCH or EGARCH model rank first and second, depending on the series, followed by the GJR model. AGARCH and the random walk models perform poorly. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19: 79–100, 1999

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*Correspondence author, Department of Economics, Finance, and Banking, Kelce College of Business, Pittsburg State University, Pittsburg, KS 66762.

■ *Kevin Bracker is an Assistant Professor at Pittsburg State University in Pittsburg, KS.*

■ *Kenneth L. Smith is a Professor at Pittsburg State University, Pittsburg, KS.*

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INTRODUCTION

Recent advances in the modeling of volatility have led to much research in various financial times series, particularly equity returns and exchange rates. However, surprisingly little research has focused on modeling the volatility of copper futures prices, given that copper is one of the most heavily traded metals. The studies that do exist on the copper market concentrate on unit roots in copper futures prices (Chowdury, 1991; Krehbiel and Adkins, 1993), testing the predictive power of copper (and aluminum) futures markets against models of price formation (Gross, 1988), tests of time-varying risk premia in forward copper prices (MacDonald and Taylor, 1989), tests of the efficient markets hypothesis/rational expectations in the copper (as well as lead, tin, and zinc) markets (MacDonald and Taylor, 1988), and tests of comovements of spot copper prices with prices of six other commodities (Pindyck and Rotemberg, 1990). Shyy and Butcher (1994) examine lead-lag relationships between the London Metals Exchange and the Shanghai Metals Exchange.

Few studies attempt to explicitly model the volatility of the copper futures market. Hardouvelis and Kim (1996) studied the volatility of copper futures contracts as it relates to margin requirements. Chang, Chen, and Chen (1990) found copper (along with platinum and silver) futures to be riskier (as measured by their standard deviations) than common stocks. Chen, Wrobleski, and Brophy (1990) apply different seasonal adjustment techniques to copper futures prices (along with futures prices from other metals, agricultural, and interest bearing commodities) to identify the best method to smooth the data.

The purpose of this paper is to fill this void. Some new modeling techniques are used to capture the volatility of the copper futures market. Specifically, an iterated cumulative sum of squares (ICSS) algorithm developed by Inclan and Tiao (1994) to detect changing volatility is applied to two copper futures prices series (open to close and close to close). The method identifies discrete subperiods of changing volatility of returns using the cumulative sum of squares of a random time series with mean zero and changing variance.

Once these episodes of changing volatility are identified, summary statistics are used to describe the volatility that exists in the subperiods. Tables Va and VIa show these statistics. The copper futures returns within the subperiods are characterized by a great deal of skewness and excess kurtosis. With this knowledge an attempt is made to judge the ability of five models to capture the movement of copper futures returns data in each of the ICSS determined subperiods. Specifically, an attempt is made to measure the ability of the naive random walk model to track movements in the copper futures market against four alternative models with long memory: one that assumes a symmetrical distribution of returns (GARCH) and three that assume asymmetrical returns (EGARCH, AGARCH, and a model attributable to Glosten, Jagannathan, and Runkle (hereafter GJR), 1993), which are explained below. These three asymmetric models are selected because (as the summary statistics below indicate) copper futures returns are negatively skewed. The symmetric GARCH model is included in the study to test whether a symmetric model captures copper futures movements as well or better than the three asymmetric models. The results indicate that the random walk model performs poorly (in terms of the in-sample root mean square error (RMSE)) in each of the subperiods when measured against the four alternative models. The results among the four alternative models are somewhat mixed, with the GARCH and EGARCH models providing the best fit to the data.

THE DATA

Initially, three sets of copper futures data are examined: open to close, close to open, and close to close returns. The sample is composed of daily data from December 31, 1974 to June 28, 1996 (5,609 observations).[1] First differences in logs of the price levels are employed in the models. Figure 1 shows the close to close data in level form, while figures 2, 3, and 4 show the close to open, open to close, and close to close returns, respectively. All three series exhibit a great deal of volatility, while showing a tendency for a constant mean. Most studies of volatility focus on close to close (usually equity) returns. Figure 4 shows the volatility inherent in close to close copper futures returns. Figures 2 and 3 also indicate a high degree of variability in close to open and open to close returns, respectively, as well.

Table I reports the summary statistics for all three return series. Like many financial time series, the distributions for the full period are negatively skewed and heavy-tailed. Because of the skewness in the data, the prior expectation is that the three asymmetric models (EGARCH, AGARCH, and GJR) provide a better fit to the data, as opposed to the symmetrical (GARCH) or the random walk models. The skewness of the data should be captured by the EGARCH, AGARCH, and GJR models, which are designed to model asymmetry.

[1]Copper futures contracts expire four times per year (March, June, September, and December). Three month contracts were used to construct a continuous series. In order to avoid any expiration effects, the new contract started a week before the expiration of the former contract.

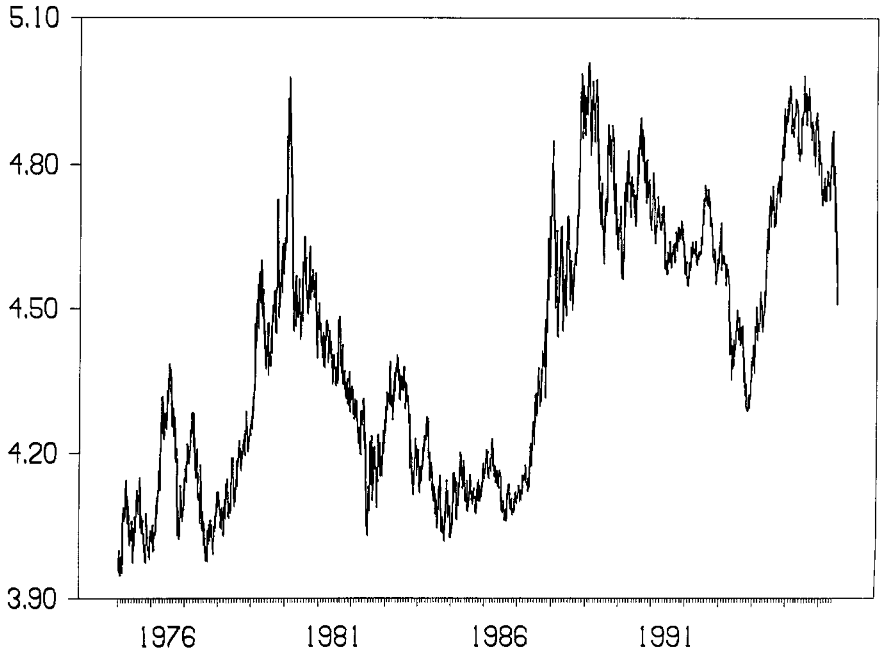

FIGURE 1
Log of daily copper futures closing prices: 1974–1996.

Of the three data series, the close to close returns exhibit the greatest variance (0.00027). As one might expect, the close to open returns are the most negatively skewed (−1.567) with the greatest excess kurtosis (20.773). This is likely due to the discrete nature of closing and opening prices. As mentioned below, the degree of nonnormality makes the close to open returns difficult to analyze.

It is, of course, imperative that the data be mean reverting. Otherwise, the variance tends to infinity as the number of observations approaches infinity, rendering the t-values unreliable and leading to spurious results. Table II reports the Dickey-Fuller and Augmented Dickey-Fuller statistics for the logs of prices, close to open, open to close, and close to close copper futures return series. Results from the table show that the test statistics for the series for the log of opening prices and the log of closing prices are not able to reject unit root. The statistics for all three first-differenced series are well below the critical values, indicating a rejection of unit root. We conclude therefore that all three series are first difference stationary and proceed with the proposed tests.

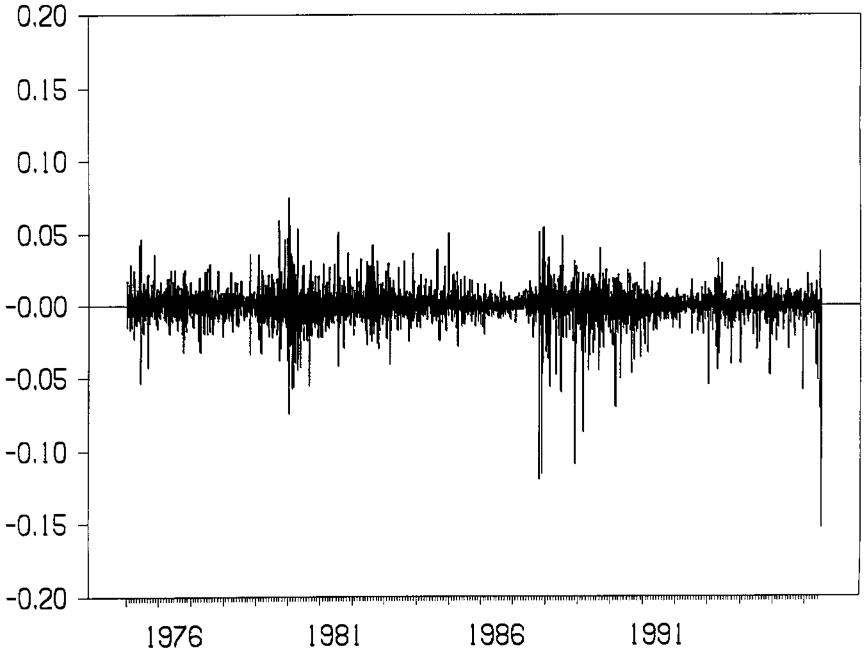

FIGURE 2
Daily copper futures close to open returns: 1974–1996.

THE MODELS AND ESTIMATION PROCEDURE

Speculative investments, such as copper, often follow a path of relative calm, interrupted by periods of greater market turbulence. This presents a problem for those attempting to model prices. The work of Bachelier (1964) modeled price movements as a random walk. Fama (1965) was able to show that these episodes of increased variance are common in financial markets. Engle (1982) captured this changing variance with the autoregressive conditional heteroscedasticity (ARCH) model.[2] Since this seminal paper many other variations of the ARCH model have been developed.

Bollerslev (1986) extended ARCH by allowing the model to include past variances as well as past forecast errors. Because of these past variances, this model is referred to as generalized ARCH (GARCH). A GARCH(1,1) model is employed and expressed as

$$\sigma_t^2 = \omega + \gamma\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{1}$$

where the restrictions $\omega > 0$, γ, and $\beta \geq 0$ are imposed to insure a positive

[2]The ARCH model has been described in many other places. Bera and Higgins (1993) provide an excellent discussion of ARCH and many of its related models.

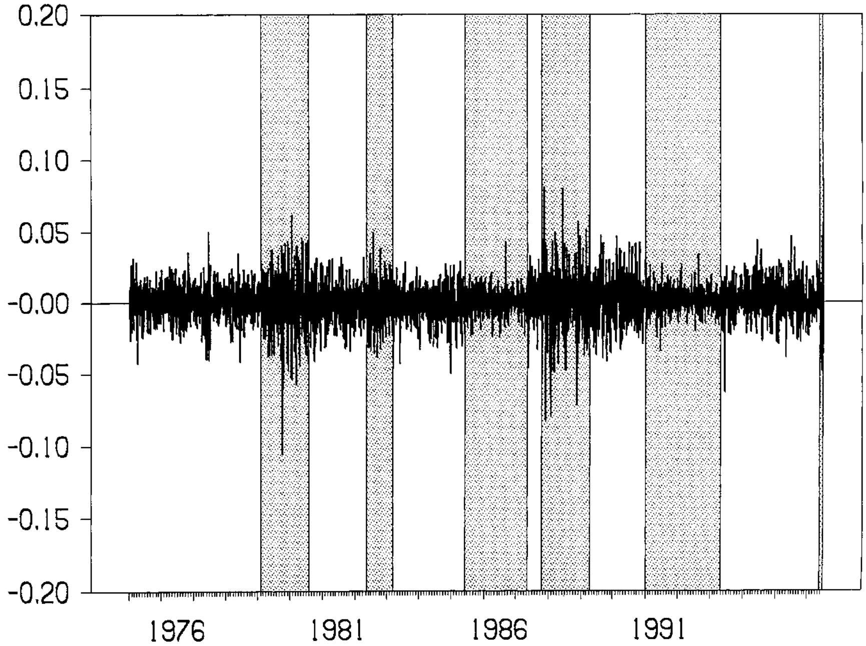

FIGURE 3
Daily copper futures open to close returns: 1974–1996.

variance. The GARCH process is analogous to an ARMA representation. Both ARCH and GARCH impose the restrictions on coefficients to ensure a positive variance. An additional restriction is that both ARCH and GARCH models assume symmetry in the distribution of asset returns. This feature has led to other models that are more likely to reflect the distributional characteristics of financial time series.

It is well known that many financial time series have nonnormal distributions. There is a well developed literature on how negative shocks increase conditional volatility (see Koutmos and Booth, 1995; Theodossiou and Lee ,1993; or Engle, 1993b) in stock market returns. These stock market returns are, like copper returns, negatively skewed with thick-tailed distributions. This suggests that these models might also be of value in capturing copper futures markets price movements.

Several models have been developed to mimic the increased volatility from negative shocks to asset returns. Nelson (1991) modeled this asymmetry using the Exponential GARCH (EGARCH) model

$$r_t = \alpha r_{t-1} + \varepsilon_t \quad (2)$$

$$\varepsilon_t|\Delta_{t-1} \sim N(0, \sigma_t^2) \quad (3)$$

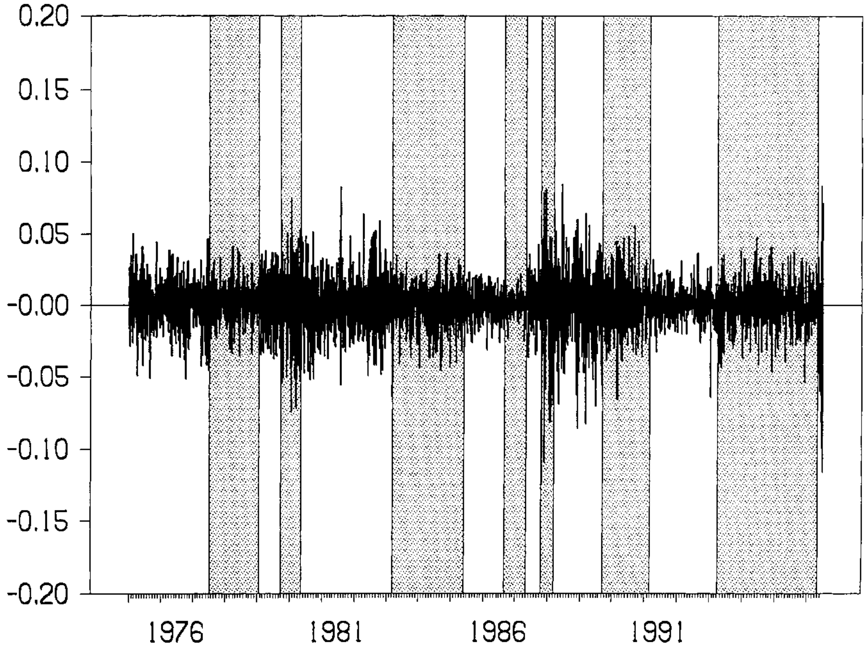

FIGURE 4
Daily copper futures close to close returns: 1974–1996.

TABLE I

Summary Statistics for Open to Close, Close to Open, and Close to Close Daily Copper Futures Returns: 31 December 1974–28 June 1996

	Open to Close	Close to Open	Close to Close
Observations	5609	5608	5608
Mean	0.00029	-0.00020	0.00010
Variance	0.00017	0.00016	0.00027
Skewness	-0.18550	-1.56737	-0.36794
Kurtosis	4.01137	20.77262	4.12270
Minimum	-0.10629	-0.15333	-0.125202
Maximum	0.08589	0.074717	0.084022
Median	0.00040	0.000000	0.000000

TABLE II

Dickey-Fuller Tests of Unit Root for the Logs of Prices, Open to Close, Close to Open, and Close to Close Daily Copper Returns.

	Dickey -Fuller	Augmented D-F
Log of Open Prices	-2.54010	-2.91269
Log of Closing Prices	-2.48739	-2.86206
Open to Close Returns	-76.55332	-29.71118
Close to Open Returns	-73.75387	-29.47106
Close to Close Returns	-76.15575	-31.66140

Notes: The Dickey-Fuller tests were conducted without lag or trend (0.01 critical value is -3.43). The augmented Dickey-Fuller Tests include five lags and a trend (0.01 critcal value is -3.96). Various specifications with lags varying from one to seven, with trend and without were tested. The results did not vary. In each case, the null of unit root could not be rejected for the undifferenced data. Unit root of the differenced data was rejected for each of the series.

$$\log \sigma_t^2 = \omega + \lambda_1 z_{t-1} + \lambda_2 (|z_{t-1}| - (2/\pi)^{1/2}) + \beta \log \sigma_{t-1}^2 \quad (4)$$

where r_t is the first difference of logs of the daily price and z_t is the normalized residual from Eq. (2). The conditional variance is estimated from Eq. (4). A negative estimated λ_1 implies that a negative shock increases the conditional variance. This provides the model with the ability to capture negative asymmetry. Additionally, an estimated positive λ_2 indicates that a shock greater than expected ($(2/\pi)^{0.5}$) also increases the conditional variance.

Alternative specifications that are designed to capture the increased volatility from asymmetric shocks are the Asymmetric GARCH (AGARCH) (Engle and Ng, 1993a) and a model developed by Glosten et al. (1993), usually referred to in the literature as the GJR model. In AGARCH the conditional variance is modeled as

$$\sigma_t^2 = \omega + \gamma(\varepsilon_{t-1} - \delta)^2 + \beta\sigma_{t-1}^2. \quad (5)$$

If δ is positive, a negative ε_{t-1} will have a larger effect on the conditional variance. Changing conditional variance in the GJR is modeled as

$$\sigma_t^2 = \omega + \gamma\varepsilon_{t-1}^2 + \delta D_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \quad (6)$$

The dummy variable, D_{t-1}, is unity when $\varepsilon_{t-1} < 0$ and zero when $\varepsilon_{t-1} \geq 0$.

If $\delta > 0$ negative shocks will have a greater impact on the conditional variance.[3]

As a benchmark, a naive, random walk model is included in this study as one of the models tested. Each of the above models are compared to the random walk. The random walk is defined as

$$\mu_t = w_t - w_{t-1}$$

$$w_t = (r_t - \bar{r})^2$$

where r_t is the return and is the average return. In a highly volatile series, such as copper futures prices, one might anticipate that this model would yield higher in-sample RMSE's than GARCH, EGARCH, AGARCH, and GJR. Indeed, this result is confirmed below by our tests.

DETECTING CHANGEPOINTS

Several papers have offered varying methods for detecting multiple changepoints in the variance of stationary series (see Hsu, 1977; Gupta and Tang, 1987; and Wichern, Miller, and Hsu, 1976). The ICSS algorithm from Inclan and Tiao (1994) is employed to detect these changepoints in the variance. Accordingly, in order to find the point in which volatilities exhibit structural breaks a sequential D_k statistic is calculated, where

$$D_k = \sqrt{\frac{T}{2}}\left(\frac{C_K}{C_T} - \frac{k}{T}\right)$$

$$C_k = \sum_{t=1}^{k} r_t^2$$

T is the total number of observations (5,609) and r_t represents the daily return. A changepoint occurs if max exceeds ± 2.225, the critical value at the 0.01% level. The series is segmented at this point and the process is repeated on the first segment. This process continues on the first subperiod until no further critical points are found. Next, return to the second subperiod (from the original critical point to the end of the series) and restart the D_k statistic from this point. If $\max\{D_k\}$ is reached and exceeds ± 2.225, the D_k statistic is restarted from the next observation. This is repeated until no critical points are found. The last critical point becomes a potential volatility changepoint. From this process, the poten-

[3]The four models were estimated by maximizing the corresponding likelihood function using RATS. The RATS code is available upon request.

tial volatility changepoints will be 0, 1, or 2. If there are zero or one potential breakpoint the process is complete. If there are two breakpoints, the entire process is repeated starting from the observation following the first potential changepoint to the second potential changepoint. This process is repeated until no further potential breakpoints are found.

Once all potential changepoints are found, they must be verified. This process breaks the sample into segments. The first segment begins at the first observation and ends at breakpoint two. The second segment starts with the first observation after changepoint one and ends with breakpoint three, and so on. The D_k statistic is calculated over each segment to confirm that the critical point coincides with the previously found potential changepoint. The breakpoint is confirmed if this is true. If not, there are two possibilities. If no critical point is found in that segment, the potential changepoint is determined to be a false positive and no true breakpoint exists. If a critical point is found but does not coincide with the previously found potential breakpoint, it becomes the new potential breakpoint. If one (or more) potential changepoint(s) is (are) not verified, another pass through is made with the new potential breakpoints. This process is repeated until all breakpoints are verified.

The decision to use a 0.01%, as opposed to the standard 5% (or even 1%), critical value may appear overly restrictive. However, this decision is related to our large sample size of 5,609 observations. There is a body of work developed by Lindley (1957), Jeffreys (1967), and Zellner and Siow (1984) that suggested standard levels of significance are not appropriate with large samples due to the possibility of a Type I error (rejecting a valid hypothesis) becoming very large. Lindley documented that it is possible to create a situation in which the statistical test rejects the null at the 5% level, while the posterior probability of the null being true, given the test statistic, is 95%. This has become known as Lindley's Paradox and is a critical issue with large sample sizes.[4] In considering standard t-tests, Zellner and Siow documented that for a sample size of 5,000 a t-statistic of 3.0 (for a two-tailed test, p. 287) would only provide a 50% likelihood that the null is not true. This t-statistic is equivalent to a significance level of 0.13%. Given this information regarding posterior probabilities and large sample sizes, a significance level of 0.01% is selected for our breakpoint analysis.[5]

[4]"Hence, for (a) fixed significance level, the likelihood of the null hypothesis increases with the sample size," (Lindley, 1957, p. 189).

[5]Results from the analysis using a 5% level are available upon request.

VISUAL INSPECTION OF THE DATA

Figure 2 shows the daily returns for the close to open returns. It is clear that the range of data points is relatively narrow. This is consistent with Table I, which indicates that the close to open returns have the smallest variance. However, Figure 2 shows that there are many spikes in the data, with more negative than positive outliers. Again, this is consistent with the high level of skewness (-1.56737) and excess kurtosis (20.77262) reported in Table I. Unfortunately, the nonnormality of the close to open returns precludes the use of the ICSS algorithm to detect changes in volatility in this series. Preliminary calculations indicate that this series would have had more than 100 breakpoints in volatility, with many subperiods being as short as three days. This, of course, rules out any estimation by any of the volatility models.[6] Our attention is therefore focused on the open to close returns and close to close returns.

Figures 3 and 4 show the open to close and close to close daily copper returns, respectively. The shaded areas indicate periods of changing volatility detected using the ICSS algorithm. The ICSS algorithm detects 11 breakpoints in the open to close series and 14 breakpoints in the close to close series. (These breakpoint dates, along with selected news events taken from the *Wall Street Journal Index,* are reported in the Appendix.)[7] Visually, one can see considerable variance within each subperiod as well as the sudden volatility jumps at the breakpoints. This is confirmation that more precision of the ICSS algorithm is necessary for modeling. Wilson, Aggarwal, and Inclan (1996, p. 325) state, ". . . the ICSS algorithm may not capture all of the variance effects; that is, there may be residual GARCH effects to that analysis. Therefore, a more complete analysis would allow for both kinds of effects." Accordingly, we test for GARCH effects, as well as breakpoints.

[6]The literature on equity returns shows that greater volatility exists at the open and closing of trading (see Stoll and Whaley, 1990; Amihud and Mendelson, 1987; and Wood, McInish, and Ord, 1985). Research by Webb and Smith (1994) showed that the greatest volatility in the Chicago Mercantile Exchange's Eurodollars futures contract occurs in the first five minutes of trading. No corresponding study exists (for which we are aware) for the copper futures market. However, the volatility, skewness, and kurtosis that exist in the close to open copper futures price series is likely due to "opening" and "closing" effects.

[7]The Appendix appears to show a dearth of new events from the *Wall Street Journal* chronology that are associated with shifts in volatility in the copper futures markets. Haugen, Talmor, and Torous (1991) showed that, for the Dow Jones Industrial Average, "a majority of the volatility changes cannot be associated with the release of significant economic information." Although no research has been carried out to test for this result for the copper futures market, it does suggest that there may be significant shifts in volatility in financial markets that are not associated with news events.

RESULTS

The full period (31 December 1974–28 June 1996) regression results for all of the estimated models are shown in Table III. All of the coefficients are highly statistically significant. The autoregressive β coefficient in each model ranges from 0.94 to 0.99. These estimates are consistent with those found in models of stock returns. The γ coefficients are likewise typically estimated to be positive in stock return models of volatility. The λ_2 coefficient in the EGARCH model is estimated to be positive. This, too, is consistent with stock return models, indicating that shocks greater than expected raise variance. These models, however, differ from the equity models in that the shock parameters are estimated to be $\delta < 0$ in the AGARCH and GJR models and $\lambda_1 > 0$ in the EGARCH model. Positive values for δ and negative values for λ_1 are consistent with Black's "leverage effect" in equity returns, which is not present in copper futures price data.[8]

The full period in-sample RMSE's are reported in Table IV. The most obvious result is the relative rank of the random walk model. As expected, in each case, the naive random walk model has the highest RMSE. Other than this, the full period RMSE rankings are mixed. EGARCH performs the best for the close to open and open to close (albeit a tie) returns, whereas the symmetrical GARCH model exhibits the lowest RMSE for the close to close returns and open to close returns (tie).

Summary statistics and RMSE's are shown in Table Va for the full period and each of the ICSS identified open to close returns subperiods. As noted above, the full period open to close returns are negatively skewed with heavy kurtosis. Thus, these full period data are clearly nonnormally distributed. As such, one might expect that the asymmetric models might yield superior modeling results. However, the symmetric GARCH model appears to do just as well as the asymmetric models. The four GARCH family models all have the same RMSE (0.000403). This suggests that with a large number of observations there is little to distinguish between these models. This study seeks to confirm that this conclusion holds for periods of changing volatility.

Subperiod Results for Open to Close Normally Distributed Returns

Table Va shows that there are several subperiods where the open to close data are not characterized by skewness or excess kurtosis. There is no

[8]Black's "leverage effect" hypothesizes that large declines in equity would raise the debt-to-equity ratio. Hentschel (1995, p. 72) discusses this effect.

TABLE III

Full Period Parameter Estimates from the GARCH, EGARCH, AGARCH, and GJR Models

	GARCH	EGARCH	AGARCH	GJR
ω	1.79e-006 (12.1605) 4.25e-006 (36.7299) 4.28e-006 (13.8392)	-0.056516 (-6.0224) -0.165813 (-14.503) -0.081283 (-6.8327)	6.58e-007 (4.37684) 1.39e-006 (8.76216) 9.71e-007 (3.36970)	7.45e-007 (5.60274) 2.09e-006 (17.0747) 1.43e-006 (5.91297)
β	0.9380000 (307.080) 0.8360000 (290.673) 0.9170000 (239.448)	0.9930601 (939.372) 0.9801712 (785.329) 0.9895256 (703.060)	0.9560000 (349.348) 0.8880000 (289.201) 0.9520000 (306.437)	0.9560000 (349.391) 0.8830000 (290.699) 0.9490000 (294.606)
γ	5.21e-002 (16.6516) 0.1410000 (38.7811) 6.82e-002 (18.6316)		4.05e-002 (15.9701) 0.1020000 (36.3383) 4.44e-002 (16.1530)	4.75e-002 (12.5319) 0.1560000 (33.4634) 5.65e-002 (13.0043)
δ			-1.67e-003 (-2.79410) -2.83e-003 (-13.8638) -2.91e-003 (-4.05998)	-1.34e-002 (-2.9057) -8.57e-002 (-15.7950) -1.89e-002 (-3.61994)
λ_1		0.1002021 (18.0742) 0.2097081 (47.5852) 0.1129616 (19.0065)		
λ_2		0.0151729 (3.81269) 0.0490179 (15.3043) 0.0174141 (4.04268)		

Notes: Within each cell the estimate for open to close returns is the top parameter, close to open is the middle parameter, and close to close is the bottom parameter. T-statistics are in parentheses.

TABLE IV

In-Sample Root Mean Square Error Each Model for the Full Period: 31 December 1974–28 June 1996

	close to open	open to close	close to close
RANDOM WALK	0.000648(5)	0.000559(5)	0.000829(5)
GARCH	0.000536(3)	0.000403(1)	0.000627(1)
EGARCH	0.000533(1)	0.000403(1)	0.000631(4)
AGARCH	0.000534(2)	0.000403(1)	0.000629(3)
GJR	0.000628(4)	0.000403(1)	0.000628(2)

evidence of nonnormality in periods four, nine, and 12. One might expect that with normally distributed data the symmetric GARCH model would exhibit the lowest RMSE. The data tend to support this contention. The GARCH model ranks first in the ninth (tie) and 12th subperiods, while ranking second in subperiod four.

The asymmetric models also perform well in these periods of normally distributed data. In subperiod nine EGARCH is tied with the GARCH and GJR models for first. The GJR model ranks first in the fourth subperiod, ties for first in the ninth subperiod, and ranks second in the 12th subperiod. AGARCH fails to converge in each of these subperiods.

Subperiod Results for Nonnormally Distributed Open to Close Returns

For the subperiods where nonnormality is rejected, the sixth subperiod exhibits the least variance (0.000059) and is characterized by the greatest excess kurtosis (2.201). The data for this subperiod are not skewed. The GARCH, EGARCH, and GJR models perform equally well (0.000121). The AGARCH model, again, fails to converge.

The first subperiod is the longest (1,066 days) in the sample. The data for this subperiod are negatively skewed (−0.196) with heavy kurtosis (1.381). In this subperiod the GARCH, EGARCH, and GJR models again perform equally well (0.000199), much better than the RW model (0.000279). The symmetric and asymmetric models have a RMSE that is 29% lower than the RW model.

Summary of the Open to Close Returns

Table Vb reports the aggregated ranks of the open to close returns. GARCH and EGARCH models ranked lowest in eight of the 12 subper-

TABLE Va

Full and Subperiod Open to Close Summary Statistics and In-Sample RMSEs

Open/Close	Full Period	1st Sub	2nd Sub	3rd Sub	4th Sub	5th Sub
N	5,609	1066	390	467	210	587
Variance	0.000171	0.000110	0.000399	0.000139	0.000242	0.000122
Skewness	-0.186^{***}	-0.196^{***}	-0.531^{***}	-0.209^{*}	0.077	-0.673^{***}
Kurtosis	4.011^{***}	1.381^{***}	1.933^{***}	0.759^{***}	0.144	2.099^{***}
naive RW RMSE	0.000559(4)	0.000279(4)	0.00107(4)	0.000307(4)	0.000460(4)	0.000279(4)
symmetric GARCH RMSE	0.000403(1)	0.000199(1)	0.000782(2)	0.000229(1)	0.000329(2)	0.000235(1)
asymmetric EGARCH RMSE AGARCH RMS GJR RMSE	 0.000403(1) 0.000403(1) 0.000403(1)	 0.000199(1) n.c. 0.000199(1)	 0.000784(3) n.c. 0.000780(1)	 0.000229(1) n.c. 0.000230(3)	 0.000331(3) n.c. 0.000324(1)	 0.000240(2) n.c. 0.000248(3)

Table Va continued

6th Sub	7th Sub	8th Sub	9th Sub	10th Sub	11th Sub	12th Sub
498	120	386	452	600	801	32
0.000059	0.000139	0.000466	0.000233	0.000060	0.000126	0.000912
0.042	-0.528^{**}	-0.254^{**}	0.121	0.027	-0.039	0.341
2.201^{***}	2.068^{***}	1.817^{***}	0.321	1.584^{***}	2.001^{***}	0.597
0.000174(4)	0.000398(4)	0.001253(4)	0.000478(4)	0.000154(4)	0.000354(4)	0.00203(3)
0.000121(1)	0.000274(2)	0.000878(2)	0.000352(1)	0.000113(1)	0.000250(1)	0.001197(1)
0.000121(1) n.c. 0.000121(1)	0.000272(1) n.c. 0.000274(2)	0.000870(1) n.c. 0.000878(2)	0.000352(1) n.c. 0.000352(1)	0.000113(1) n.c. 0.000113(1)	0.000250(1) n.c. 0.000253(3)	0.002426(4) n.c. 0.001311(2)

Notes: n.c. indicates nonconvergence. Levels of significance are *0.1 level, **0.05 level, and ***0.01 level, respectively. Ranks are in parentheses.

TABLE Vb

Open to Close Returns In-Sample RMSE Ranks from Table Va

Rank	1	2	3	4	5	6	score
RW	0	0	1	11	0	0	47(4)
GARCH	8	4	0	0	0	0	16(1)
EGARCH	8	1	2	1	0	0	20(2)
AGARCH	0	0	0	0	0	12	72(5)
GJR	6	3	3	0	0	0	21(3)

Notes: rank of 6 indicates nonconvergence. Score is the sum of the numbers in each cell multiplied by their rank.

iods. The last column of table Vb indicates a score (sum of the numbers in each cell times the corresponding rank). GARCH exhibits the lowest (16) score, followed by EGARCH (20). The GJR model exhibits the third lowest score (21). These three models appear to be the most effective for modeling of open to close copper futures returns. The random walk model does relatively poorly and the AGARCH model is of little use, it fails to converge in each of the 12 subperiods.

Subperiod Results for Normally Distributed Close to Close Returns

Table VIa shows the 15 ICSS identified subperiod results for the close to close returns. There are two subperiods (fourth and ninth) in which the close to close returns appear to be normally distributed. Again, it might be expected that with normally distributed data, the symmetric GARCH model would have the lowest RMSE. This is strictly true for the fourth subperiod, but not for the ninth. However, there are only small differences between the GARCH and EGARCH models in these subperiods. In the fourth subperiod the GARCH model reduces the RMSE over EGARCH by only 1.1%. In the ninth subperiod GARCH ranks second, behind EGARCH. In this subperiod, EGARCH reduces the RMSE over GARCH by only 1%. GJR ranks second (tie) and third, respectively, in these two subperiods.

Subperiod Results for Nonnormally Distributed Close the Close Returns

The shortest subperiod (other than the sample truncated 15th subperiod) is the 10th. It lasted only 103 days. Normality for these data is rejected

TABLE VIa

Full and Subperiod Close to Close Summary Statistics and In-Sample RMSEs

Close/Close	Full Period	1st Sub	2nd Sub	3rd Sub	4th Sub	5th Sub	6th Sub
N	5,608	661	399	172	165	736	584
Variance	0.000271	0.000213	0.000120	0.000351	0.000945	0.000304	0.000169
Skewness	-0.368***	-0.227**	0.227**	-0.090	-0.132	0.157*	-0.493***
Kurtosis	4.122***	1.155***	1.276***	-0.862**	-0.500	1.644***	0.812***
naive RW RMSE	0.000829(5)	0.000521(5)	0.000301(5)	0.000563(5)	0.001110(5)	0.000739(5)	0.000407(5)
symmetric GARCH RMSE	0.000627(1)	0.000375(4)	0.000216(3)	0.000398(3)	0.000974(1)	0.000563(1)	0.000281(2)
asymmtric EGARCH RMSE AGARCH RMSE GJR RMSE	 0.000631(4) 0.000629(3) 0.000628(2)	 0.000243(2) 0.000244(3) 0.000242(1)	 0.000137(1) 0.000216(3) 0.000138(2)	 0.000312(2) 0.000449(4) 0.000286(1)	 0.000985(2) 0.000992(4) 0.000985(2)	 0.000564(2) 0.000564(2) 0.000567(4)	 0.000279(1) 0.000286(4) 0.000281(2)

Table VIa continued

7th Sub	8th Sub	9th Sub	10th Sub	11th Sub	12th Sub	13th Sub	14th Sub	15th Sub
328	173	120	103	393	382	548	799	45
0.000010	0.000043	0.000183	0.000140	0.000532	0.000297	0.00009	0.000186	0.001536
0.206	-0.426**	-0.244	-0.821***	-0.256**	0.111	-0.397***	-0.236***	-0.596
1.411***	2.138***	0.655	1.174**	1.168***	0.675***	4.818***	1.161***	1.421*
0.000183(4)	0.000122(4)	0.000431(5)	0.003198(5)	0.001324(5)	0.000673(5)	0.000310(5)	0.000460(5)	0.003149(4)
0.000133(3)	0.000086(2)	0.000287(2)	0.001927(2)	0.000940(3)	0.000483(3)	0.000232(2)	0.000322(2)	0.002030(1)
0.000132(2) n.c. 0.000129(1)	0.000085(1) 0.000811(5) 0.000086(2)	0.000284(1) 0.000335(4) 0.000289(3)	0.001904(1) 0.001991(4) 0.001941(3)	0.000935(1) 0.000935(1) 0.000946(4)	0.000481(1) 0.000491(4) 0.000482(2)	0.000230(1) 0.000232(2) 0.000237(4)	0.000322(2) 0.000329(4) 0.000321(1)	0.083788(5) 0.002333(2) 0.002523(3)

Notes: n.c. indicates nonconvergence. Levels of significance are 0.1 level, ** 0.05 level, *** 0.01 level, respectively. Ranks are in parentheses.

TABLE VIb

Close to Close Returns In-Sample RMSE Ranks from Table VIa

Rank	1	2	3	4	5	6	score
RW	0	0	0	3	12	0	72(5)
GARCH	3	6	5	1	0	0	34(2)
EGARCH	8	6	0	0	1	0	25(1)
AGARCH	1	3	2	7	1	1	52(4)
GJR	4	5	3	3	0	0	35(3)

Notes: rank of 6 indicates nonconvergence. Score is the sum of is numbers in each cell multiplied by their rank.

by the summary statistics. For this period, EGARCH ranks first, followed by GARCH, GJR, AGARCH, and RW, respectively.

The longest subperiod is the 14th, lasting over three years (799 days). These data are negatively skewed (−0.236) with kurtosis (1.161). In this case, the GJR model exhibits the lowest RMSE (0.000321), with GARCH and EGARCH tying for the second lowest RMSE (0.000322).

The seventh subperiod returns exhibit the least variance (0.00001). This period is also characterized by excess kurtosis (1.411) and without skewness (0.206). GJR (0.000129) is the most effective model, followed by the EGARCH and GARCH (0.000132 and 0.000133, respectively) models.

It should be noted that there are two subperiods (second and fifth) that indicate statistically significant positive skewness in the close to close returns. These data are also characterized by excess kurtosis. The results for the subperiods are mixed. EGARCH ranks first in the second period, while tying with AGARCH for second in the fifth. GARCH ranks third in the second period and first in the fifth subperiod.

Summary of the Close to Close Returns

Table VIb reports the aggregate data for close to close returns. These results are similar to those of the open to close returns. EGARCH receives the lowest score (25), followed by GARCH (34), GJR (35), AGARCH (52), and random walk (72), respectively. Unlike the open to close returns, GARCH ranks second and the RW model ranks below the AGARCH model due to the improved ability of the AGARCH model to converge.

SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR FUTURE RESEARCH

The volatility of financial time series is well established. Copper futures prices are no exception. In addition to the volatility, results indicate that the data are negatively skewed with heavy kurtosis. Surprisingly little research has been done on the modelling of these time series. It is the purpose of this paper to begin to fill this gap in the literature.

The approach has been (i) to detect subperiods of changing variance; and (ii) to search for models that capture this changing variance within each subperiod for open to close and close to close copper futures returns. The full period examined is 31 December 1974 through 28 June 1996. Application of the ICSS algorithm to detect changing variance yielded 12 subperiods in the open to close returns and 15 subperiods in the close to close returns. (The extremely high degree of volatility of the close to open returns precluded any analysis of these data.)

Within each subperiod an attempt is made to determine (using the

APPENDIX

Open to Close Copper Futures Return Breakpoints As Detected by the ICSS Algorithm

Open to close copper futures return breakpoints

as detected by the ICSS algorithm

Dates	Days	Wall Street News of Copper Industry
01/31/77	1066	
07/30/80	390	
05/14/82	467	
03/04/83	210	
06/04/85	587	
05/01/87	498	
10/16/87	120	Friday before the October 19, 1987 stock market crash
04/10/89	386	
01/02/91	452	
04/21/93	600	
05/16/96	801	
06/28/96	32	

Notes: Dates are the ending days for the subperiod. Dates in bold indicate that there was also a breakpoint on the same day in the close to close series. Last column comes from the Wall Street Journal Index.

APPENDIX Continued

Open to Close Copper Futures Return Breakpoints As Detected by the ICSS Algorithm

Close to close copper futures return breakpoints

as detected by the ICSS algorithm

Dates	Days	Wall Street News of Copper Industry
07/13/77	661	Inspiration Consolidated became last major producer to lower price three cents to $0.68 a pound--07/12.
01/23/79	399	
09/20/79	172	
05/08/80	165	Commodity Exchange Inc. reduced margin requirements.
03/04/83	736	
05/30/85	584	
09/02/86	328	
05/01/87	173	
10/16/87	120	Friday before October 19, 1987 stock market crash.
03/09/88	103	Advance in copper prices attributed to supply concerns--03/08.
09/11/89	393	Copper futures prices fall. Traders' concern about drop in supplies--09/12.
02/27/91	382	
04/05/93	548	
04/26/96	799	
06/28/96	45	

Notes: Dates are the ending days for the subperiod. Dates in bold indicate that there was also a breakpoint on the same day in the open to close series. Last column comes from the Wall Street Journal Index.

RMSE) which of five models best fit the data. The five models were the random walk, the symmetric GARCH, and the asymmetric EGARCH, AGARCH, and GJR. Our results are fairly similar across both series: open to close and close to close returns. That is, the GARCH, EGARCH, and GJR models provide the best fit for both series. Results indicate that the GARCH model ranks first in modelling the open to close returns, second in modeling the close to close returns. EGARCH ranks second in the open to close returns, ranking first in the close to close returns. The GJR model ranks third in both series. The AGARCH model had great difficulty in converging in many of the subperiods. The random walk model scored lowest (behind AGARCH) for the close to close returns, and fourth (ahead of AGARCH) in the open to close series.

Previous research into the copper futures market has not focused on the volatility in the data. The results reported here indicate that any future research into the copper futures market should consider not only the volatility inherent in the data but also the skewness and excess kurtosis in the series. We find this to be true of all three copper futures series studied: close to close, open to close, and close to open returns.

The method used here is to compare in-sample RMSEs among the various models in order to *model* the data series. Future research should begin to concentrate on the out-of-sample *forecasting* ability of these (and possibly other) models.

BIBLIOGRAPHY

Amihud, Y., & Mendelson, H. (1987). Trading Mechanisms and Stock Returns: An Empirical Investigation. Journal of Finance, 43, 533–555.

Bachelier, L. (1964). Theory of Speculation. Reprinted in P. Coontner (Ed.), The random character of stock market prices, Cambridge, MA: MIT Press.

Bera, A.K., & Higgins, M.L. (1993). ARCH Models: Properties, Estimation, and Testing. Journal of Economic Surveys, 7, 305–366.

Bollerslev, T. (1986). Generalized Conditional Heteroscedasticity. Journal of Econometrics, 31, 307–327.

Chen, S.K., Wrobleski, W.J., & Brophy, D.J. (1990). An Application of Seasonal Adjustment Models to the Volatility Patterns of Futures Prices. Managerial Finance, 16, 11–18.

Chowdhury, A.R. (1991). Futures Market Efficiency: Evidence From Cointegration Tests. The Journal of Futures Markets, 11, 577–589.

Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of Variance of U.K. Inflation. Econometrica, 50, 987–1008.

Engle, R.F., & Ng, V.K. (1993a). Measuring and Testing the Impact of News on Volatility. Journal of Finance, 48, 1749–1778.

Engle, R.F., & Ng, V.K. (1993b). Statistical Models for Financial Volatility. Financial Analysts Journal, 49, 72–78.

Fama, E.F. (1965). The Behavior of Stock Market Prices. Journal of Business, 38, 34–105.

Glosten, L.R., Jagannathan, R., & Runkle, D. (1993). Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance, 48, 1779–1801.

Gross, M. (1988). A Semi-Strong Test of the Efficiency of the Aluminum and Copper Markets at the LME. The Journal of Futures Markets, 8, 67–77.

Gupta, A.K., & Tang, J. (1987). On Testing Homogeneity of Variance for Gaussian Models. Journal of Statistical Computation and Simulation, 27, 155–173.

Haugen, R.A., Tamor E., & Torous, W.N. (1991). The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns. Journal of Finance, 46, 985–1007.

Hentschel, L. (1995). All in the Family Nesting Symmetric and Asymmetric GARCH Models. Journal of Financial Economics, 39, 71–104.

Hsu, D.A. (1977). Tests for Variance Shifts at an Unknown Time Point. Applied Statistics, 26, 179–184.

Inclan, C., & Tiao, G. (1994). Use of Cumulative Sums of Squares for Retrospective Detection of Changes in Variance. Journal of the American Statistical Association, 89, 913–923.

Jeffreys, H. (1967). Theory of probability (3rd edition), London: Oxford University Press.Krehbiel, T., & Adkins, L.C. (1993). Cointegration Tests of the Unbiased Expectations Hypothesis in Metals Markets. The Journal of Futures Markets, 13, 753–763.

Krehbiel, T., & Adkins, L.C. (1989). Rational Expectations, Risk and Efficiency in the London Metal Exchange: An Empirical Analysis. Applied Economics, 21, 143–153.

Koutmos, G., & Booth, G.G. (1995). Asymmetric Volatility Transmission in International Stock Markets. Journal of International Money and Finance, 14, 747–762.

Lindley, L.V. (1957). A Statistical Paradox. Biometrika, 44, 187–192.

MacDonald, R., & Taylor, M.P. (1988). Testing Rational Expectations and Efficiency in the London Metal Exchange. Oxford Bulletin of Economics and Statistics, 50, 41–52.

Nelson, D.B. (1991). Conditional Heteroscedasticity in Asset Returns: A New Approach. Econometrica, 59, 347–370.

Pindyck, R.S., & Rotemberg, J.J. (1990). The Excess Co-movement of Commodity Prices. Economic Journal, 100, 1173–1189.

Theodossiou, P., & Lee, U. (1993). Mean and Volatility Spillovers Across Major National Stock Markets: Further Empirical Evidence. The Journal of Financial Research, 16, 337–350.

Shyy, G., & Butcher, B. (1994). Price Equilibrium and Transmission of in a Controlled Economy: A Case Study of the Metal Exchange in China. The Journal of Futures Markets, 14, 877–890.

Stoll, H., & Whaley, R. (1990). Stock Market Structure and Volatility. Review of Financial Studies, 3, 37–71.

Webb, R.I., & Smith, D.G. (1994). The Effect of Market Opening and Closing on the Volatility of Eurodollar Futures Prices. The Journal of Futures Markets, 14, 51–78.

Wichern, D.W., Miller, R.B., & Hsu, D.A. (1979). On the Likelihood Ratio Test for a Shift in Location of Normal Populations. Journal of the American Statistical Association, 74, 365–367.

Wood, R.A., McInish, T.A., & Ord, J.K. (1985). An Investigation of Transactions Data for NYSE Stocks. Journal of Finance, 3, 723–741.

Zellner, A., & Siow, A. (1984). Posterior Odds Ratios for Regression Hypotheses: General Considerations and Some Specific Results. In A. Zellner, Basic issues in econometrics, Chicago: University Chicago Press (pp. 275–305).