

Our model with non-diagonal elements in structural variance covariance matrix is given by (Elements of S have been changed to x)

$$\begin{bmatrix} \epsilon^{i^*} \\ \epsilon^y \\ \epsilon^{CP} \\ \epsilon^{MP} \\ \epsilon^{ER} \end{bmatrix} = \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 \\ x_2 & x_3 & 0 & 0 & 0 \\ x_4 & x_5 & x_6 & 0 & 0 \\ x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \end{bmatrix} \begin{bmatrix} \mu^{i^*} \\ \mu^y \\ \mu^{CP} \\ \mu^{MP} \\ \mu^{ER} \end{bmatrix} \quad (1)$$

$$\sum \mu = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{17} \\ 0 & 0 & 0 & x_{17} & 1 \end{bmatrix} \quad (2)$$

$$\sum_{\epsilon} = S \sum_{\mu} S'$$

$$\sum_{\epsilon} = \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 \\ x_2 & x_3 & 0 & 0 & 0 \\ x_4 & x_5 & x_6 & 0 & 0 \\ x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{17} \\ 0 & 0 & 0 & x_{17} & 1 \end{bmatrix} \times \begin{bmatrix} x_1 & x_2 & x_4 & x_7 & x_{12} \\ 0 & x_3 & x_5 & x_8 & x_{13} \\ 0 & 0 & x_6 & x_9 & x_{14} \\ 0 & 0 & 0 & x_{10} & x_{15} \\ 0 & 0 & 0 & x_{11} & x_{16} \end{bmatrix}$$

$$\sum_{\epsilon} = \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 \\ x_2 & x_3 & 0 & 0 & 0 \\ x_4 & x_5 & x_6 & 0 & 0 \\ x_7 & x_8 & x_9 & x_{10} + x_{11}x_{17} & x_{10}x_{17} + x_{11} \\ x_{12} & x_{13} & x_{14} & x_{15} + x_{16}x_{17} & x_{15}x_{17} + x_{16} \end{bmatrix} \times \begin{bmatrix} x_1 & x_2 & x_4 & x_7 & x_{12} \\ 0 & x_3 & x_5 & x_8 & x_{13} \\ 0 & 0 & x_6 & x_9 & x_{14} \\ 0 & 0 & 0 & x_{10} & x_{15} \\ 0 & 0 & 0 & x_{11} & x_{16} \end{bmatrix}$$

$$\sum_{\epsilon} = \begin{array}{cccccc} x_1^2 & x_1x_2 & x_1x_4 & & & \\ x_1x_2 & x_2^2 + x_3^2 & x_2x_4 + x_3x_5 & & & \\ x_1x_4 & x_2x_4 + x_3x_5 & x_4^2 + x_5^2 + x_6^2 & & & \\ x_1x_7 & x_2x_7 + x_3x_8 & x_4x_7 + x_5x_8 + x_6x_9 & & & \\ x_1x_{12} & x_2x_{12} + x_3x_{13} & x_4x_{12} + x_5x_{13} + x_6x_{14} & & & \\ & x_1x_7 & & & x_1x_{12} & \\ & x_2x_7 + x_3x_8 & & & x_2x_{12} + x_3x_{13} & \\ & x_4x_7 + x_5x_8 + x_6x_9 & & & x_4x_{12} + x_5x_{13} + x_6x_{14} & \\ x_7^2 + x_8^2 + x_9^2 + x_{10}^2 + (x_{10} + x_{11}x_{17})x_{10} + (x_{10}x_{17} + x_{11})x_{11} & & & & x_7x_{12} + x_8x_{13} + x_9x_{14} + (x_{10} + x_{11}x_{17})x_{15} + (x_{10}x_{17} + x_{11})x_{16} & \\ x_7x_{12} + x_8x_{13} + x_9x_{14} + (x_{15} + x_{16}x_{17})x_{10} + (x_{15}x_{17} + x_{16})x_{11} & & & & x_{12}^2 + x_{13}^2 + x_{14}^2 + (x_{15} + x_{16}x_{17})x_{15} + (x_{15}x_{17} + x_{16})x_{16} & \end{array}$$

$$C_{54}(1) = E_{51}(1)S_{14} + E_{52}(1)S_{24} + E_{53}(1)S_{34} + E_{54}(1)S_{44} + E_{55}(1)S_{54} = 0 \quad (3)$$

which can be written as

$$C_{54}(1) = E_{51}(1) \times 0 + E_{52}(1) \times 0 + E_{53}(1) \times 0 + E_{54}(1) \times x_{10} + E_{55}(1) \times x_{15} = 0 \quad (4)$$

these two gives us

$$x_{10} = \frac{-E_{55}}{E_{54}} \times x_{15} \quad (5)$$

Our objective is to solve $\sum_{\epsilon} = S \sum_{\mu} S'$. \sum_{ϵ} has 15 free parameters. Our $S \sum_{\mu} S'$ is symmetric and can be mapped to \sum_{ϵ} but it has 17 variables which can't be estimated uniquely. One long run restriction helps in reducing the variable by 1 and using one more long run restrictions, we can identify the model. We can solve it using nonlinear estimation. isn't it?

Why can't we do this?