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Re-examining the asymmetric predictability of conditional variances: The role of sudden changes in variance

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Abstract

The existence of "spillover effects" in financial markets is well documented and multivariate time series techniques have been used to study the transmission of conditional variances among large and small market value firms. Earlier research has suggested that volatility surprises to large capitalization firms are a reliable predictor of the volatility of small capitalization firms. A related line of research has examined how regime shifts in volatility may account for a considerable amount of the persistence in volatility. However, these studies have focused on univariate modeling and many have imposed regime changes on a priori grounds. This paper re-examines the asymmetry in the predictability of the volatilities of large versus small market value firms allowing for sudden changes in variance. Our method of analysis extends the existing literature in two important ways. First, recent advances in time series econometrics allow us to detect the time periods of sudden changes in volatility of large cap and small cap stocks *endogenously* using the iterated cumulated sums of squares (ICSS) algorithm. Second, we directly incorporate the information obtained on sudden changes in volatility in a Bivariate GARCH model of small and large cap stock returns. Our findings indicate that accounting for volatility shifts considerably reduces the transmission in volatility and, in essence, removes the

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spillover effects. We conclude that ignoring regime changes may lead one to significantly *overestimate* the degree of volatility transmission that actually exists between the conditional variances of small and large firms.

JEL classification: F3
Keywords: Volatility; Capitalization; Bivariate GARCH; ICSS algorithm

1. Introduction

The idea that conditional volatility is transmitted across markets is not new. In fact, the existence of "spillover effects" in financial markets is well documented. [2] Moreover, multivariate time series techniques that allow for time-varying volatility have been used to study the transmission of conditional variances among the returns of a number of assets including large and small market value firms. Earlier research has suggested that volatility surprises to large capitalization firms are a good predictor of the volatility of small capitalization firms (Conrad et al., 1991b). A related line of research has examined how regime shifts in volatility may account for a considerable amount of the persistence in volatility (e.g., Lastrapes, 1989; Lamoureux and Lastrapes, 1990). However, these studies have focused on univariate modeling and many have imposed regime changes on a priori grounds. This paper re-examines the asymmetry in the predictability of the volatilities of large versus small market value firms with specific emphasis on the role that sudden changes in variance may play in the transmission process. Our method of analysis extends the existing literature in two important ways. First, recent advances in time series econometrics allow us to detect the time periods of sudden changes in volatility of large cap and small cap stocks *endogenously* using the iterated cumulated sums of squares (ICSS) algorithm (Aggarwal et al., 1999). The technique is an improvement over previous methods as it can locate the time of change as well as the duration. Second, we directly incorporate the information obtained on sudden changes in volatility in a Bivariate GARCH model of small and large cap stock returns. [3] Our findings show that accounting for volatility shifts considerably reduces the transmission in volatility and, in essence, removes the spillover effects. The evidence suggests that failing to account for regime changes may lead one to significantly *overestimate* the degree of volatility transmission that actually exists between the conditional variances of small

[2] Since the well-known articles of Shiller (1981a,b), much attention has focused on determining the extent to which inter-relationships among various asset markets exist. Examples include papers by Hamao et al. (1990), King and Wadhwani (1990), Ng et al. (1991), Cheung and Kwan (1992), Engle and Susmel (1993), Lin et al. (1994), King et al. (1994), Kim and Rogers (1995), Karolyi (1995), and Ewing et al. (2002), to name just a few.

[3] Aggarwal et al. (1999) incorporated the sudden change information in a univariate model of emerging market stock returns. To our knowledge, we are the first to extend this to the multivariate case. This may be particularly useful in the study of volatility transmission such as in the case of the small cap–large cap debate.

and large firms. The results have important implications for building accurate asset pricing models, forecasting volatility of stock returns, managing market capitalization exposure and furthering our understanding of stock markets.

2. Brief background on small–large cap relationships

It is often purported that portfolios comprised of small cap stocks outperform those of large cap stocks on a risk-adjusted basis and, in light of this, Reinganum (1999) documents the importance of managing exposure to market capitalization. Further highlighting the importance of capitalization, Daniel and Titman (1998) argue that expected returns are determined by characteristics such as size rather than by return patterns. Moreover, Lo and MacKinlay (1990) documented the now well-known finding that large cap returns lead small cap returns but the reverse is not true. It appears that this large cap–small cap unidirectional or asymmetrical relationship is not limited to returns, either. A number of studies have documented the transmission of volatility from large to small cap portfolios. Generally speaking, these findings have been explained using arguments based on thin trading, asymmetric transactions costs, differential information hypothesis and the like.

Motivated by Ross' (1989) claim that the variance of asset price changes is directly related to the rate of flow of information, Conrad et al. (1991b) examined the transmission of conditional variances between large and small market value firms. They looked at weekly return data from the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX) over the period of 1962–1988 and found evidence of an asymmetric pattern in the transmission of conditional variances. Volatility surprises to large cap firms could be used to reliably predict the volatility in returns of small market value firms.

Chelley-Steeley and Steeley (1996) estimated the effects of volatility transmission between UK firms of differing sizes. Their analysis used a two-step procedure where they first estimated volatility and then used this estimate to examine how and to what extent transmission may occur. Their results actually indicated bi-directional feedback of conditional variances. However, these results should be interpreted with caution as they did not simultaneously estimate all the parameters of the model. Grieb and Reyes (2002) also examined data for the UK. Using the bivariate GARCH model they found some evidence to corroborate Chelley-Steeley and Steeley (1996). Moreover, they were able to conclude that volatility persistence was larger in small caps than in large cap stocks.

In another related study, Henry and Sharma (1999) examined the relationship between firm size and equity volatility for two portfolios comprised of Australian firms. Their analysis consisted of both univariate and bivariate GARCH models and demonstrated that conditional variance is related to firm size. Also examining volatility in a univariate framework, Koutmos et al. (1994) examined and compared volatility persistence in 10 international stock markets differentiated by market capitalization. They concluded that small capitalization markets exhibit higher volatility persistence than large capitalization markets.

3. Background on volatility and regimes

Two stylized facts about financial asset prices are that they tend to be processes that are integrated of order one, I(1), and their corresponding returns are leptokurtic (i.e. fat tails). The former property is often taken as evidence consistent with the weak-form efficient markets hypothesis. In particular, shocks to (the mean of) an I(1) series are permanent, whereas shocks to the first-difference of the series are transitory in nature. The latter property often manifests as volatility clustering and suggests that the conditional variance of the return series may not be constant. This time-varying property implies that shocks to the series affect volatility for several, if not many, periods into the future. While a number of techniques have been used to model volatility, the autoregressive conditional heteroskedasticity (ARCH) model developed by Engle (1982), and later generalized by Bollerslev (1986), is by far the most popular method used for analyzing high frequency financial time series data. [4] A common finding is that shocks to volatility are extremely persistent, implying that "current information remains important for the forecasts of the conditional variances for all horizons" (Engle and Bollerslev, 1986, p. 27).

Lastrapes (1989) and Lamoureux and Lastrapes (1990) argue that persistence in volatility is *overestimated* when standard (G)ARCH models are applied to a series with underlying sudden changes in variance. For instance, consider a one time increase in unconditional variance in which low-magnitude shocks cluster in the initial period (i.e., low variance regime) and high magnitude shocks cluster in the period after the shift (i.e., high variance regime). Low magnitude shocks will thus appear to be followed by similar low variance values while high magnitude shocks will be followed by correspondingly high variance values. Thus, it will appear that the impact of shocks on volatility does not die out quickly. If regime shifts can affect persistence in "own" volatility, then it is possible that allowing for regime shifts in individual series may also affect the persistence of volatility across two series. Consequently, a finding of asymmetry may be due to a mis-measurement in persistence. Additionally, the asymmetry may be affected because the regime shift itself may be "common" across the series. Furthermore, the presence of volatility regimes gives rise to volatility clustering, that is, periods of low (high) volatility are followed by more periods of low (high) volatility. A GARCH model that ignores structural shift(s) will suggest high persistence of shocks on volatility. Extending this line of reasoning to the multivariate GARCH case where the variance shifts can be linked across variables, perhaps triggered by common events (e.g., a recession, etc.), may result in rather strong transmission among second moments. [5]

[4] See Bollerslev et al. (1992) for a detailed survey.

[5] This is a simple intuitive explanation. However, Mikosch and Starica (2004) give a detailed theoretical explanation as to why ignoring shifts in unconditional variance will yield higher volatility persistence in the GARCH model. They support their claim using simulations and data on US equity returns. Their research made a significant contribution by providing a solid theoretical explanation for this phenomenon.

The shifts in volatility could be due to regime/structural changes that are caused by political, social or economic events. However, markets may very well anticipate some events in advance and sometimes respond with a time lag, so we do not expect events to correlate to changes in sudden variance on any specific day. [6] Additionally, Ross (1989) showed that volatility in asset returns depends upon the rate of information flow, suggesting that information from one market can be incorporated into the volatility generating process of the other market. Since flow of information and the time used in processing that information can possibly vary depending on the market size, one should expect different volatility patterns across markets based on capitalization. However, this flow of information may be influenced by structural changes that can affect, among other things, the intensity of the flow, its direction, or even its origin. Also, volatility spillovers are usually attributed to changes in common information and cross-market hedging, which may simultaneously alter expectations across markets. Fleming et al. (1998) show how cross-market hedging and sharing of common information can transmit volatility across markets over time. Engle et al. (1990) argue that volatility in one financial market is transmitted to other markets like a ''meteor shower.'' Due to these reasons, the same event can quite possibly affect different markets at different time points and thus making it difficult to isolate those events which 'cause' volatility breaks. We make no attempt in this paper to absolutely identify the causes of the sudden changes and instead focus on identifying the time periods of sudden changes themselves.

Hamilton and Susmel (1994) examined stock returns using a (G)ARCH model and made an argument similar to that of Lastrapes (1989). Their analysis employed a Markov switching model to account for structural/regime changes. Unlike Lastrapes (1989), the structural breaks were determined by the data. Cai (1994) developed a Markov–ARCH model that endogenizes structural breaks in order to examine the issue of volatility persistence in the Treasury bill market. [7] To get reliable parameter estimates of the conditional variance equation, regime shifts should be incorporated in the standard GARCH model. Hamilton (1994) argues that a good model should account for structural or regime shifts.

[6] An avenue of future research could be to use intra-daily data and conduct separate event studies on the break points found here and see what events 'cause' volatility breaks. However, looking at the newspaper reporting for significant events surrounding the break points should be done cautiously as there is a bias in media to always attribute reasons for specific stock market volatility, even in cases where the market may very well be 'adjusting' to some prior news event.

[7] Markov switching models have been used in the literature to study structural shifts in the mean. Since we are interested in shifts in variance, a possible alternative would be the model given by Cai (1994) extended to GARCH model. However, the model with GARCH parameters cannot be estimated without making very restrictive assumptions on the data generating process as mentioned by Cai (1994). In this paper, we are interested in detecting the shift points in variance and not the probabilities associated with those shifts. The ICSS algorithm performs that task without making any restrictive assumptions. Also, as explained in the methodology section, the ICSS algorithm identifies the shifts by properly recognizing outliers and 'masking effects' which makes it an excellent tool for retrospective detection of break points in variance.

4. The empirical methodology

In this section, we discuss how in this paper we examine the transmission of volatility between large cap and small cap stocks within a bivariate GARCH model framework allowing for endogenously determined sudden changes in variance. Specifically, we compare and contrast results from two versions of the model, namely, one that ignores volatility regime changes and one that controls for them.

4.1. Detecting points of sudden changes in variance

The methodology used in detecting sudden discrete changes in variance is based on the ICSS algorithm given by Inclan and Tiao (1994). Their analysis focuses on detecting a variance change in a given time series due to a sudden shock which changes the variance until the series experiences a future shock. Let ε_t be a series with zero mean and with unconditional variance σ^2. Let the variance within each interval be given by $\sigma_j^2, j = 0,1,\ldots,N_T$, where N_T is the total number of variance changes in T observations, and $< \kappa_1 < \kappa_2 < \cdots < \kappa_{NT} < T$ are the change points,

$$\sigma_t^2 = \tau_0^2 \qquad \text{for } 1 < t < \kappa_1, \tag{1a}$$

$$\sigma_t^2 = \tau_1^2 \qquad \text{for } \kappa_1 < t < \kappa_2, \tag{1b}$$

$$\vdots \qquad\qquad \vdots$$

$$\sigma_t^2 = \tau_{NT}^2 \qquad \text{for } \kappa_{NT} < t < T. \tag{1c}$$

To detect the number of changes in variance and the time point of each variance shift, a cumulative sum of squares is used. Let us denote $C_k = \sum_{t=1}^{k} \varepsilon_t^2,\ k = 1,\ldots,T$, as the (mean-centered) cumulative sum of squares from the first observation to the kth point in time. Define the D_k statistic as follows:

$$D_k = (C_k/C_T) - k/T, \quad k = 1,\ldots,T \quad \text{with } D_0 = D_T = 0. \tag{2}$$

Note that if the series contains no changes in variance, then the D_k statistic will oscillate around zero and if plotted against k will resemble a horizontal line. However, if the series contains variance changes, then it will plot as a drift from zero. Significant changes in variance are detected based on critical values obtained from the distribution of D_k under the null hypothesis of constant variance. Specifically, the null hypothesis of homogeneous variance is rejected if the maximum absolute value of D_k is greater than the critical value. Define k^* to be the value at which $\max_k|D_k|$ is reached. If $\max_k\sqrt{(T/2)}|D_k|$ falls outside the predetermined boundary, then k^* is taken as the time point of a variance change in the series. The term $\sqrt{(T/2)}$ is required to standardize the distribution.

Following Aggarwal et al. (1999), a critical value of 1.36 is used, which is the 95th percentile of the asymptotic distribution of $\max_k\sqrt{(T/2)}|D_k|$. Therefore, in the D_k plot the upper and lower boundaries are set at ± 1.36. A change point in variance is identified if it falls outside these boundaries. However, for a series in which there are multiple change points, detection is made difficult due to the "masking effects."

To overcome this problem Inclan and Tiao (1994) designed an algorithm that looks at different pieces of the series for identification of the change points in variance. The ICSS algorithm works by evaluating D_k over different periods of time and those different periods are determined by the break points identified by the D_k plot.

4.2. Bivariate GARCH model

The first step in the Bivariate GARCH procedure is to identify the best-fitting specification of the (mean) return series using standard Box–Jenkins techniques. [8] The following mean return equation was chosen and estimated for each series:

$$R_{it} = \mu_i + \varphi_i R_{it-1} + \varepsilon_{it}, \tag{3}$$

where R_{it} is the return on portfolio i between time $t-1$ and t, μ_i is a long-term drift coefficient, and ε_{it} is the error term for the return on portfolio i at time t. Eq. (3) was then tested for the existence of autoregressive conditional heteroskedasticity (ARCH) using the test described in Engle (1982, p. 1000). The mean equation for both the large cap and the small cap series exhibited evidence of ARCH effects. [9] Thus, estimation of a GARCH-class model is appropriate. However, since we are interested also in the possibility of volatility transmission between the two market segments, as well as volatility persistence within each segment, we employ a variant of the multivariate GARCH model capable of handling this type of phenomena.

There are two popular parameterizations for the multivariate GARCH model. [10] The VECH model, introduced by Bollerslev et al. (1988), is given by

$$\mathrm{vech}(H_t) = A_0 + \sum_{j=1}^{q} B_j \mathrm{vech}(H_{t-j}) + \sum_{j=1}^{p} A_j \mathrm{vech}\,(\varepsilon_{t-j}\varepsilon'_{t-j}), \tag{4}$$

where $\varepsilon_t = H_t^{1/2}\eta_t$, $\eta_t \sim \text{iid } N(0, I)$. Here, H_t is the conditional variance matrix, and the notation $\mathrm{vech}(X_t)$ implies a vector formed by stacking the columns of matrix X_t. The total number of estimated elements for the variance equation in our bivariate case is 21. Note that during estimation all elements must be constrained to be

[8] See Mills (1999) for an overview of how one fits an ARMA model using Box–Jenkins techniques.

[9] The test statistic is distributed χ^2 with degrees of freedom equal to the number of restrictions. Specifically, we found significant first-order ARCH effects in each series. This finding suggests that past values of volatility can be used to predict current volatility. Additionally, the coefficient for lagged small cap return regressed on large cap return was -0.06 with a p-value of 0.06 and the coefficient for lagged large cap return regressed on small cap return was 0.02 with a p-value of 0.59. In other words, we did not find any significant spillover effects in mean across the variables, so only the "own" lagged returns were used in the analysis. This is particularly important as mis-specifying the mean equation may lead to incorrect estimation of the variance equation.

[10] Other parameterizations for multivariate GARCH models include a constant correlation model, which drastically reduces the number of estimated parameters by assuming constant correlations between the variables across time. Some earlier studies (Karolyi, 1995; Bollerslev, 1990) used this assumption; however, Longin and Solnik (1995) contend that the assumption of constant correlations over time does not hold for the equity markets. Also, the assumption is rather restrictive in the sense that it does not allow for cross effects in the variance equation. Our approach incorporates less restrictive assumptions.

positive in order to guarantee a positive semi-definite covariance matrix, which is a rather formidable task.

A viable alternative to the above is the BEKK [11] model of Engle and Kroner (1995). This model incorporates quadratic forms in such a fashion that the covariance matrix will be positive semi-definite, a requirement that is needed to ensure that the estimated variances are non-negative.

The BEKK parameterization for the Bivariate GARCH(1, 1) model can be written as

$$H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A, \tag{5}$$

where in the bivariate case, C is a 2×2 lower triangular matrix with three parameters, and B is a 2×2 square matrix of parameters. The latter matrix depicts the extent to which current levels of conditional variances are related to past conditional variances. A is also a 2×2 square matrix of parameters and measures the extent to which conditional variances are correlated with past squared errors (i.e., deviations from the mean). The elements of A capture the effects of shocks or events on volatility (conditional variance). In our case, the total number of estimated parameters is 11.

The conditional variance for each equation can be expanded for the bivariate GARCH (1, 1) as follows:

$$h_{11,t+1} = c_{11}^2 + b_{11}^2 h_{11,t} + 2b_{11}b_{12}h_{12,t} + b_{21}^2 h_{22,t} + a_{11}^2\varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + a_{21}^2\varepsilon_{2,t}^2, \tag{6}$$

$$h_{22,t+1} = c_{12}^2 + c_{22}^2 + b_{12}^2 h_{11,t} + 2b_{12}b_{22}h_{12,t} + b_{22}^2 h_{22,t} + a_{12}^2\varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + a_{22}^2\varepsilon_{2,t}^2. \tag{7}$$

Eqs. (6) and (7) reveal how shocks and volatility are transmitted over time and across the small and large cap stocks. [12]

The following likelihood function is maximized assuming normally distributed errors:

$$L(\theta) = -T\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\ln|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t), \tag{8}$$

where T is the number of observations and θ represents the parameter vector to be estimated. Numerical maximization techniques were used to maximize this non-linear log likelihood function. Following the recommendations of Engle and Kroner (1995), several iterations were performed with the simplex algorithm to obtain the

[11] The acronym BEKK is used in the literature as an earlier draft of the published paper was written by Baba et al. (1990). Conrad et al. (1991b) used BEKK in their analysis.

[12] Note that the coefficient terms in Eqs. (6) and (7) are a non-linear function of the estimated elements from Eq. (5). A first-order Taylor expansion around the mean was used in order to calculate the standard errors for these coefficient terms. [See, for example, Kearney and Patton (2000) for more on the use of this methodology to obtain standard errors.]

initial conditions. The BFGS algorithm was then employed to obtain the final estimate of the variance–covariance matrix and corresponding standard errors. [13]

4.3. GARCH model with sudden changes

Lastrapes (1989) and Lamoureux and Lastrapes (1990) argue that when standard (G)ARCH models are applied to series with sudden changes in variance then the results indicate more persistence in volatility than is actually the case. In order to obtain reliable parameter estimates of the conditional variance equation, regime shifts should be incorporated in the standard GARCH model. Hamilton and Susmel (1994) use a switching ARCH (SWARCH) model to introduce regime changes. Our study uses the GARCH model controlling for sudden changes in variance which have been identified by the ICSS algorithm as follows. In particular, we introduce a set of dichotomous variables to the model given in (5) such that

$$H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t'A + \sum_{i=1}^{n} D_i'X_i'X_iD_i. \tag{9}$$

Eq. (9) differs from Eq. (5) by the inclusion of the last term. D_i is a 2×2 square diagonal matrix of parameters and X_i is a 1×2 row vector of volatility regime control variables, and n is the number of break points found in variance. In this paper, we found three break points in each series, so n was equal to 3. First (second) element in X_i row vector represents the dummy for first (second) series. If the first series undergoes a volatility break at time t, then the first element will take a value of zero before time t and a value of one from time t onwards. This specification of dummy variables was used to allow for a common shift among large and small cap returns in variances as well as separate shifts depending on the detected breaks points obtained from using the ICSS algorithm. Consequently, we do not rule out the possibility of a relation between the shifts in the variance of the large (small) securities' portfolio and the variance of the small (large) portfolio. Aggarwal et al. (1999) and Malik (2003) used this methodology to detect shifts in volatility of stock returns in emerging markets and exchange rates, respectively, and concluded that volatility persistence is overestimated if these endogenously determined breakpoints are ignored. Our study fundamentally differs from these univariate analyses as we are interested in the transmission of volatility between two series where either one or both of them may have experienced sudden changes in variance. In the presence of these changes, it is possible that the transmission embedded in the bivariate nature of the small cap–large cap relationship may be affected. Detecting the points and duration of changes in variance as well as accounting for them in the bivariate volatility transmission model is the focus of this paper.

[13] Quasi-maximum likelihood estimation was used and robust standard errors were calculated by the method given by Bollerslev and Wooldridge (1992). All calculations were performed using RATS version 4.01 (Regression Analysis of Time Series).

5. A description of the large firm–small firm data series

We calculated weekly Wednesday-close returns from daily price data obtained from the Center for Research in Security Prices (CRSP) for both New York and American Stock Exchanges. CRSP Stock File Capitalization Decile Indices are used for NYSE and AMEX. In these market segment indices all securities excluding American Depositary Receipts for both exchanges are ranked according to capitalization each rebalancing period and then divided into deciles. The portfolios are rebalanced each year using the market capitalization at the end of the previous year. The market value-weighted portfolio is constructed each year using all issues listed on both exchanges with available shares outstanding and valid prices in the current and previous periods. The small cap series consists of the smallest (decile 1) group and the large cap series the largest (decile 10) group based on market capitalization. The data are daily closing values for the period from January 1, 1988 to December 31, 2001. Weekly returns are used in the analysis and should not be subject to potential biases such as the bid–ask effect, non-trading days, etc. that might arise when using daily returns. Portfolio returns are used rather than individual security returns as the latter contain dramatically more noise (Conrad et al., 1991a). Moreover, the multivariate GARCH model cannot be estimated for a large number of securities as it is computationally cumbersome. In the case of a holiday occurring on a Wednesday, the value on the previous day of trading was used to calculate the return. Consistent with the literature, the price series in levels were found to have a unit root while the return series was found to be stationary. [14]

Table 1 provides descriptive statistics for the small cap and large cap return series. Consistent with conventional wisdom, both mean return and standard deviation are higher for small cap stocks than for large cap stocks. Additionally, each series exhibits evidence of excess kurtosis while small cap returns also exhibit autocorrelation. Figs. 1 and 2 present plots of the two return series. There are two points of interest revealed in these plots. First, it appears that for each series there are periods of high (low) volatility that are followed by periods of high (low) volatility. This observation, known as volatility clustering, is consistent with the statistical finding of excess kurtosis (i.e., "fat tails") and suggests that the variance of the series may be time-varying. Second, the two series often exhibit periods of relative tranquility (low volatility) and periods of heightened volatility around the same time. Certainly, no definitive statements should be made regarding time-varying volatility or volatility spillovers from a cursory visual examination of the series. Thus, we proceed to a more formal time series examination that allows for the possibility that the variance of each series is changing over time and that is capable of capturing volatility spillovers between the small and large cap stock returns.

[14] Results of unit root tests are available on request.

Table 1
Descriptive statistics

Return series	Mean	SD	Skewness	Kurtosis	Q(16)
Small cap	0.00316	0.02403	1.12918	10.47969	104.88*
Large cap	0.00253	0.02007	−0.56752	5.003154	8.5786

Notes: All statistics are for weekly returns. Q(16) is the Ljung–Box statistic for the presence of autocorrelation. * Denotes significance at 5% level. The sample period is from the first week of January 1988 to the last week of December 2001. There are 729 weekly observations.

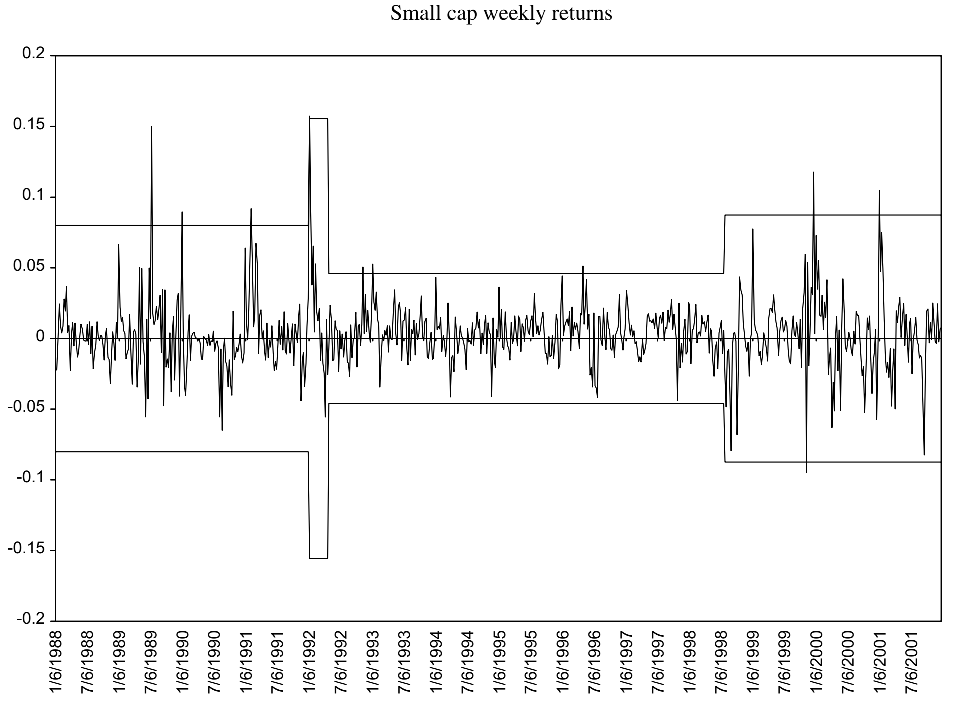

Fig. 1. Small cap weekly returns. (*Notes:* The sample period is from first week of January 1988 to last week of December 2001. Bands at ±3 standard deviations, change points estimated using ICSS algorithm.)

6. Discussion of the empirical results

While our intention is to re-examine the predictability of conditional variance of small and large cap firms allowing for volatility regime changes, it is helpful to first consider the baseline case of the bivariate GARCH model without regime controls. The results of estimating this bivariate GARCH model with BEKK parameterization for each variance equation are reported in Table 2. Our findings indicate that volatility (conditional variance) in small firm returns is directly affected by its own volatility and by the volatility in the large firm returns. Higher levels of conditional

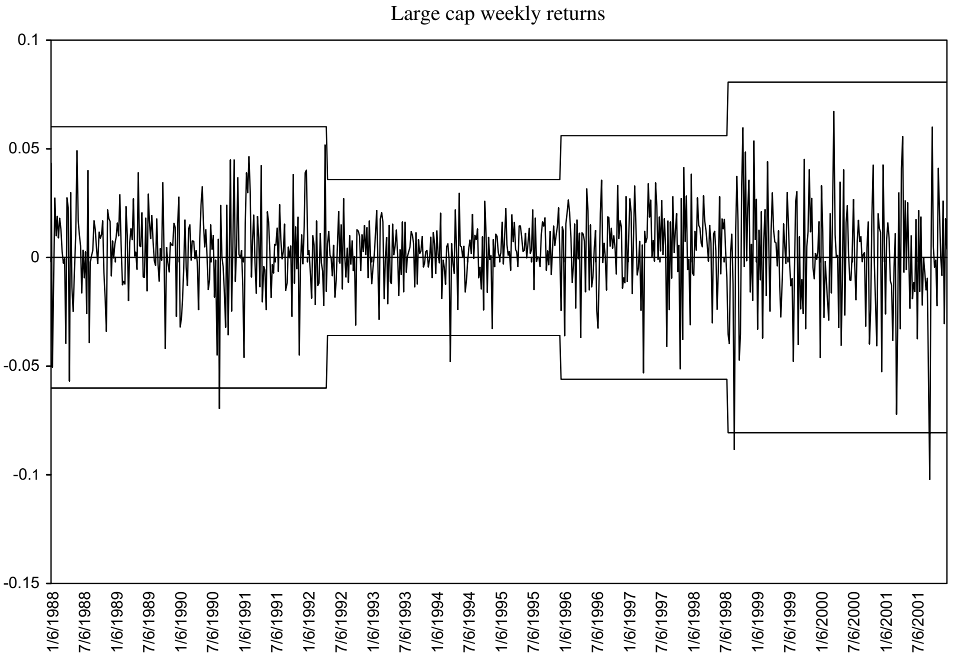

Fig. 2. Large cap weekly returns. (*Notes:* The sample period is from first week of January 1988 to last week of December 2001. Bands at ±3 standard deviations, change points estimated using ICSS algorithm.)

Table 2
Results of bivariate GARCH model ignoring volatility regime changes

Small cap conditional variance equation:

$$h_{11,t+1} = 1.34\times 10^{-5} + 0.82h_{11,t} - 1.68h_{12,t} + 0.86h_{22,t} + 0.15\varepsilon_{1,t}^2 + 0.08\varepsilon_{1,t}\varepsilon_{2,t} + 0.01\varepsilon_{2,t}^2$$

(2.06) (5.96) (−12.48) (4.88) (3.10) (2.04) (1.08)

Large cap conditional variance equation:

$$h_{22,t+1} = 1.20\times 10^{-6} + 0.005h_{11,t} - 0.15h_{12,t} + 1.04h_{22,t} + 4.0\times 10^{-4}\varepsilon_{1,t}^2 - 0.007\varepsilon_{1,t}\varepsilon_{2,t} + 0.03\varepsilon_{2,t}^2$$

(0.18) (0.21) (−0.42) (6.51) (0.25) (−0.50) (4.79)

Volatility persistence comparison:

Small cap return	0.25
Large cap return	0.92

Notes: h_{11} denotes the conditional variance for the small cap return series and h_{22} is the conditional variance for the large cap return series. Directly below the estimated coefficients, and given in parentheses, are the corresponding *t*-values. The mean equations included a constant term and a lagged return term. Three break points were detected by the ICSS algorithm for each return series. Results for the mean equations are not reported for the sake of brevity but are available upon request. The AR(1) term in both mean equations was significant at conventional levels.

volatility in the past are associated with higher conditional volatility in the current period (note the positive and significant coefficients on h_{11} and h_{22}). Moreover, the coefficient for the covariance term in the conditional variance equation for small cap returns is statistically significant. This latter finding implies indirect volatility transmission through the covariance term from large cap returns to small cap returns. Thus, consistent with Conrad et al. (1991b), the results in Table 2 indicate significant direct and indirect transmission of volatility from large firms to small firms. These results also indicate that volatility in small cap returns is affected by shocks originating in small cap (i.e., the estimated coefficients on ε_1^2 is significant). Looking at the results for the large cap firms, we see that volatility (conditional variance) in large cap returns is only affected by its own volatility and its own 'news' (as measured by its own mean return error term). News originating in small caps or small cap volatility does not affect large cap volatility. Table 2 also provides estimates of the persistence in volatility for each return series. [15] Large cap returns exhibit considerably more volatility persistence than small caps. Similar asymmetric effects were found by Conrad et al. (1991b).

However, the early work of Lastrapes (1989) and later work by Aggarwal et al. (1999) clearly showcase a role for volatility regimes in affecting persistence. Applying the ICSS algorithm to our series resulted in the identification of three distinct break points for each return series (see Table 3). There appear to be some commonalities in the variance shifts across both series perhaps due to some major event that would trigger a variance change in both series. For example, in July 1998 the two series experienced a sudden increase in volatility *simultaneously*. This is the time in which Russia experienced tremendous political and financial turmoil and, consequently, this may have contributed to the volatility in equity markets. The regimes are shown in Figs. 1 and 2 by the use of bands at ± 3 standard deviations. Given that the variance of each series has experienced sudden changes and distinct regimes exist we proceed to the estimation and discussion of the bivariate GARCH model that controls for these shifts in volatility.

The results for the bivariate GARCH model controlling for regime changes are shown in Table 4 and are in sharp contrast to the results presented in Table 2. [16] The incorporation of the sudden changes in variances effectively removes the asymmetric effects found earlier. The interpretation is that large stocks do not affect small stocks once endogenously determined regime shifts are incorporated in the model. Moreover, even news in large cap stocks does not affect its own volatility. These results are consistent with the finding of Lastrapes that (bivariate) GARCH effects could be a manifestation of structural and/or regime shifts. Additionally, the

[15] Analogous to the univariate GARCH case, the persistence of volatility in the multivariate GARCH model is computed by taking the sum of coefficients of lagged variances, covariances, squared error terms and cross-product of error terms.

[16] The Ljung–Box test for serial correlation in the cross product between standardized residuals is a useful diagnostic test for misspecification in the variance equation. The Ljung–Box statistic (equal to 22.06 at 16 lags) was found to be insignificant at the 10% level implying that no autocorrelation remains in the residuals of the estimated model.

Table 3
Sudden changes in variance

	No. of change points	Time period	SD
Small cap	3	Jan 1, 1988–Jan 1, 1992	0.02671
		Jan 8, 1992–Apr 22, 1992	0.05182*
		Apr 29, 1992–July 22, 1998	0.01531
		July 29, 1998–Dec 31, 2001	0.02911*
Large cap	3	Jan 1, 1988–Apr 22, 1992	0.02003
		Apr 29, 1992–Dec 13, 1995	0.01196
		Dec 20, 1995–July 22, 1998	0.01868*
		July 29, 1998–Dec 31, 2001	0.02688*

Notes: * Denotes period of increased volatility. Time periods detected endogenously by the ICSS algorithm. The sample period is from first week of January 1988 to last week of December 2001.

Table 4
Results of bivariate GARCH model controlling for endogenously determined volatility regime changes

Small cap conditional variance equation:

$$h_{11,t+1} = 3.1 \times 10^{-5} + 0.72h_{11,t} + 0.01h_{12,t} + 8.6 \times 10^{-5}h_{22,t} + 0.19\varepsilon_{1,t}^2 + 0.04\varepsilon_{1,t}\varepsilon_{2,t} + 0.002\varepsilon_{2,t}^2$$

(1.59) (11.00) (0.20) (0.10) (2.72) (0.51) (0.24)

Large cap conditional variance equation:

$$h_{22,t+1} = 9.19 \times 10^{-5} + 0.005h_{11,t} - 0.12h_{12,t} + 0.66h_{22,t} + 0.01\varepsilon_{1,t}^2 + 0.024\varepsilon_{1,t}\varepsilon_{2,t} + 0.01\varepsilon_{2,t}^2$$

(2.25) (0.95) (−1.63) (3.23) (0.90) (0.69) (0.38)

Volatility persistence comparison:

Small cap return	0.98
Large cap return	0.59

Notes: h_{11} denotes the conditional variance for the small cap return series and h_{22} is the conditional variance for the large cap return series. Directly below the estimated coefficients, and given in parentheses, are the corresponding *t*-values. The mean equations included a constant term and a lagged return term. Three break points were detected by the ICSS algorithm for each return series. Results for the mean equations are not reported for the sake of brevity but are available upon request. The AR(1) term in both mean equations was significant at conventional levels.

estimated coefficients in the variance equations are quite different in the model with volatility regime controls. For example, the estimated coefficient which determines how large firms lagged conditional variance affects its own conditional variance drops in value from 1.04 in the model that ignores sudden changes to 0.66 in the model that takes sudden changes in variance into account. It is worth noting that controlling for regimes has reduced the volatility in persistence in the case of large caps. At first glance, it may seem just the opposite case for small caps; however, many of the coefficients are now insignificant and thus interpretation of volatility persistence by summation is not meaningful.

The importance of considering changes in variance is further supported by the likelihood ratio statistic (LR). The likelihood ratio statistic is calculated as $\text{LR} = 2[L(\Theta_1) - L(\Theta_0)]$ where $L(\Theta_1)$ and $L(\Theta_0)$ are the maximum log likelihood values obtained from the GARCH models with and without changes, respectively. This statistic is asymptotically χ^2 distributed with degrees of freedom equal to the number of restrictions from the more general model (with changes) to the more parsimonious model (without changes). In our case, $\text{LR} = 2(5100.42 - 5095.17) = 10.5$, thus we can clearly reject the null of no variance change even at the 1% significance level. In the next section, we note that appropriately estimating the variance–covariance matrix has important implications for asset pricing, risk management and portfolio selection.

Finally, in order to compare our results to those of Conrad et al. (1991b), we repeated the analysis using the same sample period as theirs, namely, 1962–1988. We obtained virtually identical mean and variance equations as reported in their paper. Our results for this sample period were very similar to those we reported above. In particular, in the model without regime controls and in line with Conrad et al. (1991b), we found that volatility in large caps has a significant impact on the volatility of small cap firms, while the reverse does not hold. However, this asymmetric effect vanishes when the ICSS identified regime shifts are incorporated in the model. [17]

7. Some economic implications of the findings

Our results have important economic implications. For example, let us consider the problem of computing the optimal fully invested portfolio holdings subject to a no-shorting constraint. Portfolio managers are often faced with this problem when deriving their optimal portfolio holdings. In order to avoid forecasting expected returns, we assume here that the expected returns are zero, making the problem equivalent to estimating the risk minimizing portfolio weights. Let

$$w_t = \frac{h_{22t} - h_{12t}}{h_{11t} - 2h_{12t} + h_{22t}}.$$

Assuming a mean-variance utility function, the optimal portfolio holdings of the small-firm portfolio are:

$$w_t^* = 0 \text{ if } w_t < 0, \quad w_t \text{ if } 0 \leqslant w_t \leqslant 1 \text{ and } 1 \text{ if } w_t > 1.$$

The optimal holdings of the large firm portfolio are $1 - w_t^*$. As shown in Panel A of Table 5, the model that ignores sudden changes in variance gives an average optimal weight of 0.14 while the model with the regime change controls gives an average of 0.71. The correlation between the two is fairly low (0.41). The optimal portfolio will depend on the covariance model chosen, meaning that portfolio managers would have to be very careful which covariance model they select because model choice matters considerably.

[17] Results are not reported for the sake of brevity but are available on request.

Table 5
Portfolio comparisons from the estimated models

	Bivariate GARCH	Bivariate GARCH with volatility regime controls
Panel A: Optimally fully invested small firm portfolio weights		
Average	0.1409	0.7151
Correlation 0.4127		
Panel B: Optimal risk minimizing large firm hedge ratios		
Average	0.2074	0.6588
Correlation 0.1185		

Note: Correlation reports the correlation coefficient between the Bivariate GARCH model with and without the volatility regime controls.

For another example, consider the problem of calculating the dynamic risk minimizing hedge ratio using each of the two specifications of the model, that is, one that ignores regime shifts and one that accounts for them. Kroner and Sultan (1993) have used the constant correlation version of the multivariate GARCH model and Baillie and Myers (1991) have used the VECH version to calculate these ratios. Kroner and Ng (1998) show that different parameterizations of multivariate GARCH models may result in different hedge ratios. However, here we are able to document that the same basic bivariate GARCH model may result in different values depending on whether or not regime shifts are accounted for in the model. Kroner and Sultan (1993) show that to minimize the risk of a portfolio an investor should short \$ β of the large-firm portfolio that is \$1 long in the small-firm portfolio, where the 'risk minimizing hedge ratio' β is given as

$$\beta_t^* = \frac{h_{12,t}}{h_{22,t}}.$$

We show the estimated 'risk minimizing hedge ratio' (β) in Panel B of Table 5 for our bivariate GARCH models with and without the regime dummy variables, respectively. The average for the model that ignores the sudden changes in variance is 0.2074 compared to 0.6588 for the model that controls for the volatility regimes. Note also that the correlation has a strikingly low value of 0.11. Clearly, the choice of the model may drastically affect the estimated hedge ratio and ignoring regime shifts will seriously lead to wrong hedging decisions. For example, when holding a long position for \$100 in the small portfolio, investors will short \$20 using the model without regime controls and \$65 for the model controlling for regimes.

8. Summary and concluding remarks

In this paper, we have re-examined the asymmetry in the predictability of the volatilities of large versus small market value firms by taking into account the role played by sudden changes in variance. Our method of analysis has extended the existing literature in two important ways. First, recent advances in time series econometrics allow us to detect the time periods of sudden changes in volatility of large cap and small cap stocks *endogenously* using the iterated cumulated sums of squares

(ICSS) algorithm. Second, we directly incorporate the information obtained on sudden changes in variance in a bivariate GARCH model of small and large cap stock returns.

Our findings indicate that accounting for volatility shifts considerably reduces the transmission in volatility and, in essence, removes the spillover effects. We conclude that ignoring regime changes may lead one to significantly *overestimate* the degree of volatility transmission that actually exists between the conditional variances of small and large firm returns. [18] The results have important implications for building accurate asset pricing models, forecasting volatility of stock returns, managing market capitalization exposure and furthering our understanding of stock markets. Finally, Conrad et al. (1991b) correctly interpret their results as suggesting "that any model of time-varying expected returns and volatilities needs to take into account the asymmetric predictability of the mean and variance of different capitalization companies." We would augment their suggestion by saying that any such model should also take into account the effects of possible regime shifts and sudden changes in variance.

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[18] An avenue of future research could be to perform some Monte Carlo exercises to see if a multivariate data generating process that includes independent volatility regimes could deceive a simple multivariate GARCH model into detecting spurious spillover effects.

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