Overview

This procedure generates simulated out-of-sample forecasts, forecast errors, and tests of equal mean square error (MSE) and encompassing for a pair of nested models (the first model is a restricted version of the second) of a scalar predictand.

Details on notation, forecast tests, critical values, etc., are available from:

> McCracken, Michael W., 2004, "Asymptotics for Out of Sample Tests of Granger Causality," manuscript, University of Missouri. Available at: http://www.missouri.edu/~mwmd4f/
>
> Clark, Todd. E., and Michael W. McCracken, 2001, "Tests of Equal Forecast Accuracy and Encompassing for Nested Models," *Journal of Econometrics* 105 (Nov.), pp. 85-110.

Users of the tests of equal MSE should cite the McCracken paper. Users of the critical values for tests of forecast encompassing should cite the Clark and McCracken paper.

> An Excel file with complete tables of critical values is available from either
> http://www.missouri.edu/~mwmd4f/
> or
> http://www.kansascityfed.org/econres/staff/tec.htm

Notation/setup

y_t : scalar random variable to be predicted

$x_{1,t}$ = set of regressors in the restricted model (model 1)

$x_{2,t} = (x'_{1,t}, x'_{22,t})'$: set of regressors in the unrestricted model (model 2)

k_2 : number of variables in model 2 not in model 1

R = number of observations used for estimation of the model from which the first forecast is formed

P = number of forecasts

π = P/R (technically, the asymptotics are based on the limit of P/R as both P and R go to infinity). In any given application with given values of P and R, P/R can't equal 0. However, for P small relative to R, some researchers may wish to consider critical values based on $\lim_{P,R\_\infty}(P/R) = 0$ – that is, use critical values based on an asymptotic approach which differs from that associated with the critical values for $\pi > 0$. As noted below, the asymptotic distributions associated with $\pi = 0$ differ from those associated with $\pi > 0$.

recursive scheme: forecasting models estimated with more data as forecasting moves forward in time

rolling scheme: forecasting models estimated with a moving window of most recent R observations as forecasting moves forward in time

fixed scheme: forecasting models estimated just once, with first R observations; same coefficient estimates used to generate all forecasts

1-step ahead forecast errors from models 1 and 2: $\hat{u}_{1,t+1} = y_{t+1} - x'_{1,t+1}\hat{\beta}_{1,t}$ and $\hat{u}_{2,t+1} =$

$y_{t+1} - x'_{2,t+1}\hat{\beta}_{2,t}$

Note: To simplify notation going forward, for any variable z_{t+1}, let $\sum_t z_{t+1}$ denote the summation $\sum_{t=R}^{T} z_{t+1}$.

Note: As discussed in Clark and McCracken (2001), among other sources, the tests are one-sided: the only rejections of interest under the alternative are those in the right tail.

<u>Calculation/definition of test statistics for equal MSE</u>

Let $d_{t+1} = \hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2$ and $\bar{d} = P^{-1}\sum_t d_t$ = $MSE_1 - MSE_2$

$$\text{MSE-t} = P^{1/2}\frac{\bar{d}}{\sqrt{P^{-1}\sum_t (d_{t+1} - \bar{d})^2}} = P^{1/2}\frac{P^{-1}\sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)}{\sqrt{P^{-1}\sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)^2 - \bar{d}^2}}$$

$$\text{MSE-F} = P \times \frac{P^{-1}\sum_{t=R}^{T} (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)}{P^{-1}\sum_{t=R}^{T} \hat{u}_{2,t+1}^2} = P \times \frac{MSE_1 - MSE_2}{MSE_2}$$

Note: An alternative t-statistic for equal MSE, referred to as MSE-REG by Clark and McCracken (2001), is calculated as the t-statistic associated with the coefficient _ from the OLS regression $\hat{u}_{1,t+1} - \hat{u}_{2,t+1} = \alpha(\hat{u}_{1,t+1} + \hat{u}_{2,t+1}) +$ error term. As described in McCracken (2004), the MSE-REG test was proposed by Granger and Newbold (1977). The asymptotic distribution of the MSE-REG test is the same as that of the MSE-t test (i.e., the appropriate critical values are those reported for the MSE-t test).

Calculation/definition of test statistics for forecast encompassing

Let $c_{t+1} = \hat{u}_{1,t+1}(\hat{u}_{1,t+1} - \hat{u}_{2,t+1}) = \hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1}$ and $\bar{c} = P^{-1}\sum_t c_t$.

$$\text{ENC-t} = P^{1/2}\frac{\bar{c}}{\sqrt{P^{-1}\sum_t (c_{t+1} - \bar{c})^2}} = P^{1/2}\frac{P^{-1}\sum_t (\hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1})}{\sqrt{P^{-1}\sum_t (\hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1})^2 - \bar{c}^2}}.$$

$$\text{ENC-NEW} = P \cdot \frac{\bar{c}}{MSE_2} = \mathrm{P} \cdot \frac{\mathrm{P}^{-1}\sum_t (\hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1})}{\mathrm{P}^{-1}\sum_t \hat{u}^2_{2,t+1}}.$$

Note: An alternative t-statistic for forecast encompassing, referred to as ENC-REG by Clark and McCracken (2001), is calculated as the t-statistic associated with the coefficient _ from the OLS regression $\hat{u}_{1,t+1} = \alpha(\hat{u}_{1,t+1} - \hat{u}_{2,t+1}) +$ error term. The asymptotic distribution of the ENC-REG test is the same as that of the ENC-t test (i.e., the appropriate critical values are those reported for the ENC-t test).